Class : IX
Subject: Maths

Chapter # 2
Real and Complex Numbers

1. <u>Natural numbers:</u> The numbers 1, 2, 3 ... which we use for counting certain objects are called natural numbers and denoted by "N". i.e $N = \{1, 2, 3 \ldots\}$

2. <u>Whole Numbers:</u> If we includes "0" in the set of natural numbers, the resulting set is the set of whole numbers, denoted by "W" i.e $W = \{0, 1, 2, 3 \cdots\}$

3. <u>Integers:</u> The set of integers consist of Positive integers, 0 and negative integers and denoted by Z.
i.e $Z = \{ \cdots -3, -2, -1, 0, 1, 2, 3 \cdots \}$

4. <u>Rational Numbers:</u> All numbers of the form P/q, where P, q are integers and $q \neq 0$ are called rational numbers.

5. The set of rational numbers is denoted by Q.
i.e $Q = \{P/q \mid P, q \in Z \wedge q \neq 0\}$

5. <u>Irrational Numbers:</u>
The numbers which cannot be expressed as Quotient of integers are called irrational numbers and denoted by Q'
i.e $Q' = \{x / x \neq P/q \wedge P, q \in Z, q \neq 0\}$

e.g $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$ and e are called irrational numbers.

6. <u>Real Numbers:</u> The union of the set of rational numbers and Irrational numbers is known as the set of real numbers. It is denoted by "R" i.e $R = Q \cup Q'$

7. <u>Terminating Decimal Fraction:</u> The decimal fraction in which there are finite number of digits in its decimal Part is called a terminating decimal fraction 0.4, 0.375 etc

8. <u>Recurring and Non Terminating Decimal Fraction:</u>
The decimal fraction in which some digits are repeated again and again in the same order in its decimal Part is called a recurring decimal fraction.

e.g $\frac{2}{9} = 0.222 \ldots$ and $\frac{4}{11} = 0.3636 \cdots$

(1) identify which of the following are rational and irrational numbers.

i) $\sqrt{3}$ irrational

ii) $\frac{1}{6}$ Rational

iii) π irrational

iv) $\frac{15}{2}$ Rational

v) 7.25 Rational

vi) $\sqrt{29}$ irrational

(2) Convert the following statements are true and which are false?

i) $\frac{17}{25}$

$= \boxed{0.68}$

$25\overline{)170}$ = 0.68
150
200
200
×

ii) $\frac{19}{4}$

$= \boxed{4.75}$

$4\overline{)19}$ = 4.75
16
30
28
20
20
×

iii) $\frac{57}{8}$

$= \boxed{7.125}$

$8\overline{)57}$ = 7.125
56
10
8
20
16
40
40
×

(2)(iv) $\frac{205}{18}$

$= \boxed{11.3888\ldots}$

Recurring decimal

If we round off to 3·dp

Ans is $\boxed{11.389}$

$18\overline{)205}$ = 11.3888...
18
25
18
70
54
160
144
160
144
16

v) $\frac{5}{8}$

$= \boxed{0.625}$

$8\overline{)50}$ = 0.625
48
20
16
40
40
×

vi) $\frac{25}{38}$

$= 0.65789\ldots$

If we round off to 4 d·p

Ans $\boxed{0.6579}$

$38\overline{)250}$ = 0.65789
228
220
190
300
266
340
304
360
342
18

(3) which of the following statements are true and which are false

i) $\frac{2}{3}$ is an irrational number $\boxed{F}$

ii) π is an irrational number $\boxed{T}$

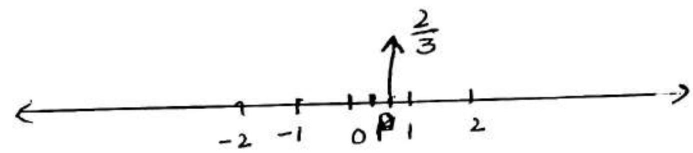
(iii) $\frac{1}{9}$ is a terminating fraction $\boxed{F}$

iv) $\frac{3}{4}$ is a terminating fraction $\boxed{T}$

v) $\frac{4}{5}$ is a recurring fraction $\boxed{F}$

(4) Represent the following numbers On the number line.

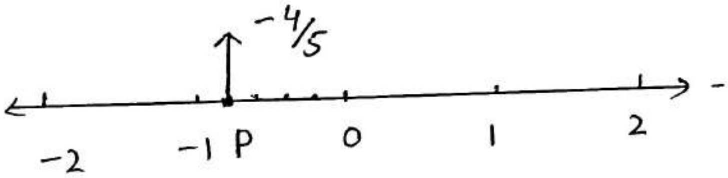
i) $\frac{2}{3}$ (because Denominator is 3)



So Divide into 3 equal Parts b|w 0 and 1.

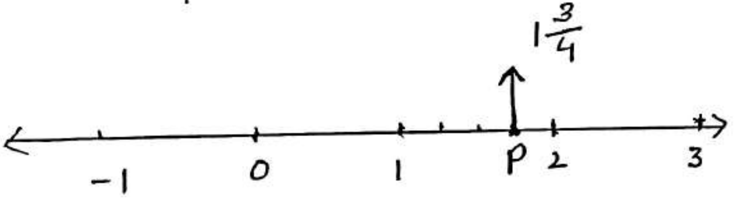
ii) $-\frac{4}{5}$ (because Denominator is 5

So divide into 5 Equal Parts b/w 0 and -1



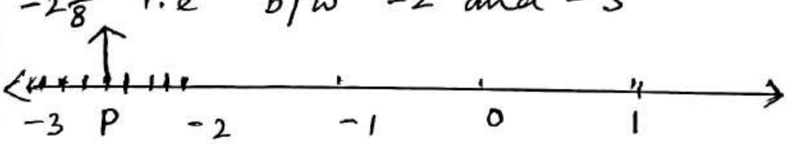
iii) $1\frac{3}{4}$ (because denominator is 4

So divide into 4 Equal Parts b/w 1 and 2



iv) $-2\frac{5}{8}$

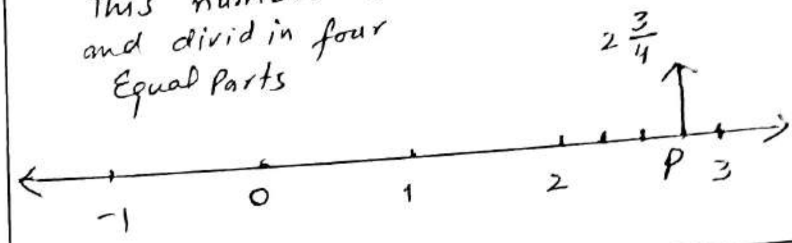
This number will be after -2 i.e b/w -2 and -3



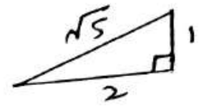
Divide into 8 equal Parts

v) $2\frac{3}{4}$

This number will be b/w 2 and 3 and divid in four Equal Parts

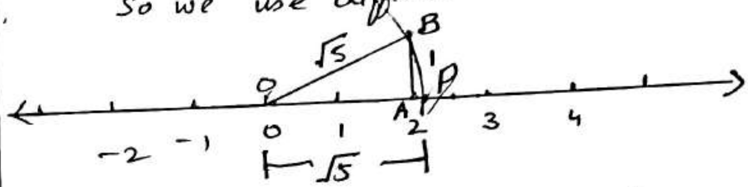


vi) $\sqrt{5} = \sqrt{(2)^2 + (1)^2}$



By Paythagoras theorem

$\sqrt{5}$ is an irrational number So we use different method



By drawing an arc with centre at O and radius $OB = \sqrt{5}$ we get the Point P representing $\sqrt{5}$ on the number line.

(5) give a rational number b/w $\frac{3}{4}$ and $\frac{5}{9}$

$$\text{Number} = \frac{\left(\frac{3\times 9}{4\times 9} + \frac{5\times 4}{9\times 4}\right)}{2}$$

$$= \frac{\left(\frac{27+20}{36}\right)}{2}$$

$$= \frac{47}{36} \div \frac{2}{1}$$

$$= \frac{47}{36} \times \frac{1}{2} = \boxed{\frac{47}{72}}$$

Q) Express the following recurring decimal as the rational number p/q where p, q are integers and $q \neq 0$

i) $0.\overline{5}$

Let $x = 0.\overline{5}$

$x = 0.5555....$

when one digit is repeating then multiply by 10 on both sides

$10x = 10 \times 0.5555\cdots$

$= 5.5555\cdots\cdots$

$10x = 5 + 0.555\cdots\cdots$

$10x = 5 + x$

$10x - x = 5$

$9x = 5$

$$\boxed{x = \frac{5}{9}}$$

ii) $0.\overline{13}$

Let $x = 0.\overline{13}$

$x = 0.131313\cdots$

when two digits are repeating then multiply by 100

$100x = 0.131313\cdots \times 100$

$100x = 13.131313\cdots$

$100x = 13 + 0.131313\cdots$

$100x = 13 + x$

$100x - x = 13$

$99x = 13$

$$\boxed{x = \frac{13}{99}}$$

iii) $0.\overline{67}$

Let $x = 0.\overline{67}$

$x = 0.676767\cdots$

Multiply by 100 on both sides

$100x = 0.676767\cdots \times 100$

$100x = 67.6767\cdots$

$100x = 67 + 0.676767\cdots$

$100x = 67 + x$

$100x - x = 67$

$99x = 67$

$$\boxed{x = \frac{67}{99}}$$

Exercise 2.2

Q1 Identify the Property used in the following

i) $a+b = b+a$ (Commutative w.r.t addition)

(ii) $(ab)c = a(bc)$ (associative w.r.t multiplication)

iii) $7\times 1 = 7$ (Multiplicative identity)

iv) $x>y$ OR $x=y$ OR $x<y$ (Trichotomy)

(v) $ab = ba$ (Commutative w.r.t multiplication)

(vi) $a+c = b+c \Rightarrow a=b$ (Cancellation Property of addition

(vii) $5+(-5) = 0$ (additive inverse)

viii $7\times\frac{1}{7} = 1$ (multiplicative inverse)

ix) $a>b \Rightarrow ac = bc$ $(c>0)$ (multiplicative Property)

(2) Fill in the following blanks by Stating the Properties of real Numbers use.

$3x + 3(y-x)$

$= 3x + 3y - 3x$ (Distributive of multiplication over subtraction)

$= 3x - 3x + 3y$ (Commutive w.r.t addition)

$= 0 + 3y$ (additive inverse)

$= 3y$ (additive identity)

(3) Give the name of Property used in the following

i) $\sqrt{24} + 0 = \sqrt{24}$ (additive identity)

(ii) $-\frac{2}{3}\left(5+\frac{7}{2}\right) = -\frac{2}{3}(5) + \left(-\frac{2}{3}\right)\left(\frac{7}{2}\right)$ (Distributive Property)

iii) $\pi + (-\pi) = 0$ (additive inverse)

iv) $\sqrt{3} - \sqrt{3}$ is a real number. (Closure Property)

v) $\left(-\frac{8}{5}\right)\left(-\frac{5}{8}\right) = 1$ (Multiplicative inverse)

(1) Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.

i) $\sqrt[3]{-64} = (-64)^{1/3}$

ii) $2^{3/5} = \sqrt[5]{2^3}$

iii) $-7^{1/3} = \sqrt[3]{-7}$

iv) $y^{-2/3} = \sqrt[3]{y^{-2}}$

(2) Tell whether the following statements are true or false?

i) $5^{1/5} = \sqrt{5}$ $\boxed{F}$

ii) $2^{2/3} = \sqrt[3]{4}$ $\boxed{T}$

iii) $\sqrt{49} = \sqrt{7}$ $\boxed{F}$

iv) $\sqrt[3]{x^{27}} = x^3$ $\boxed{F}$

(3)

(i) $\sqrt[3]{-125} = (-125)^{1/3}$

R.W

5	125
5	25
5	5
	1

$= (-5^3)^{1/3}$

$= -5^{\cancel{3}\times\frac{1}{\cancel{3}}}$

$\sqrt[3]{-125} = (-5)^1$

(3) ii) $\sqrt[4]{32} = (32)^{1/4}$

R.W

2	32
2	16
2	8
2	4
2	2
	1

$= (2\times2\times2\times2\times2)^{1/4}$

$= (2)^{1/4}\,(2^4)^{1/4}$

$= (2)^{1/4}\,(2)$

$= 2\left(\sqrt[4]{2}\right)$

(3) iii) $\sqrt[5]{\frac{3}{32}}$

$= \frac{(3)^{1/5}}{(32)^{\frac{1}{5}}} = \cancel{\frac{(3)^{1/5}}{(2^1\times2^4)^{1/5}}}$

$= \frac{\sqrt[5]{3}}{(2^5)^{\frac{1}{5}}}$ $(32 = 2^5)$

$= \frac{\sqrt[5]{3}}{2}$

(3)(iv) $\sqrt[3]{\frac{8}{-27}}$

we know
$8 = 2^3$
$27 = 3^3$

$= \frac{(8)^{1/3}}{(-27)^{1/3}}$

$= \frac{(2^3)^{1/3}}{(-3^3)^{1/3}}$

$= \boxed{-\frac{2}{3}}$

Exercise 2.4

laws of Exponents / indices:

if $a, b \in R$ and m, n are positive integers, then

I $a^m \cdot a^n = a^{m+n}$

II $(a^m)^n = a^{mn}$

III $(ab)^n = a^n b^n$

IV $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

V $\frac{a^m}{a^n} = a^{m-n}$

VI $a^0 = 1$

VII $a^{-n} = \frac{1}{a^n}$

1/ Use the laws of exponent to simplify

$$\frac{(243)^{-\frac{2}{3}} \ (32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$$

$$\frac{(3^5)^{-\frac{2}{3}} \ (2^5)^{-\frac{1}{5}}}{\sqrt{[(14)^2]^{-1}}}$$

$$\frac{\left(3^{-\frac{10}{3}}\right)\left(2^{-5 \times \frac{1}{5}}\right)}{\left[(14)^{-\cancel{2}}\right]^{\frac{1}{\cancel{2}}}}$$

3	243
3	81
3	27
3	9
3	3
	1

$\sqrt{\ } = \frac{1}{2}$

$$\frac{(3)^{-\frac{10}{3}} \ (2)^{-1}}{(14)^{-1}}$$

Note: whenever there is a -ve Power, you change into +ve Power by changing the side.

$$= \frac{\overset{7}{\cancel{(14)}}}{(3)^{\frac{10}{3}} \ \cancel{(2)^1}}$$

$$= \frac{7}{(3^{10})^{\frac{1}{3}}} = \frac{7}{(3^9 \cdot 3^1)^{\frac{1}{3}}}$$

$$= \frac{7}{3^{\frac{9 \times 1}{3}} \cdot 3^{\frac{1}{3}}}$$

Note

$\sqrt[3]{\ } = \frac{1}{3}$

$$= \frac{7}{3^3 \sqrt[3]{3}}$$

$$= \boxed{\frac{7}{27\sqrt[3]{3}}}$$

ii) $(2x^5y^{-4})(-8x^{-3}y^2)$

$$= -16\, x^{5-3} \ y^{-4+2}$$

$$= -16\, x^2\, y^{-2}$$

$$= \boxed{\frac{-16x^2}{y^2}}$$

(1) iii)

$$\left(\frac{x^{-2}\, y^{-1}\, z^{-4}}{x^{4}\, y^{-3}\, z^{0}}\right)^{-3}$$

$$= \frac{x^{6}\, y^{3}\, z^{12}}{x^{-12}\, y^{9}\, z^{0}}$$

Now collect power of x on base x, power of y on base y, similarly on z

$$= x^{6+12}\, y^{3-9}\, z^{12-0}$$

$$= x^{18}\, y^{-6}\, z^{12}$$

$$= \boxed{\frac{x^{18}\, z^{12}}{y^{6}}}$$

(4)

$$\frac{(81)^{n}\cdot 3^{5} - (3)^{4n-1}\cdot(243)}{(9)^{2n}\cdot(3^{3})}$$

$$= \frac{(3^{4})^{n}\cdot 3^{5} - (3)^{4n-1}\cdot(3^{5})}{(3^{2})^{2n}\cdot 3^{3}}$$

$$= \frac{3^{4n+5} - 3^{4n-1+5}}{3^{4n}\cdot 3^{3}}$$

$$= \frac{3^{4n}\cdot 3^{5} - 3^{4n+4}}{3^{4n}\cdot 3^{3}}$$

$$= \frac{3^{4n}\cdot 3^{5} - 3^{4n}\cdot 3^{4}}{3^{4n}\cdot 3^{3}}$$

$$= \frac{\cancel{3^{4n}}\cdot\cancel{3^{3}}\left[3^{2} - 3^{1}\right]}{\cancel{3^{4n}}\cdot\cancel{3^{3}}}$$

$$= 9 - 3 = \boxed{6}$$

(2) Show that

$$\left(\frac{x^{a}}{x^{b}}\right)^{a+b} \times \left(\frac{x^{b}}{x^{c}}\right)^{b+c} \times \left(\frac{x^{c}}{x^{a}}\right)^{c+a} = 1$$

$$L.H.S = (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c-a}$$

$$= (x)^{a^2-b^2} \times (x)^{b^2-c^2} \times (x)^{c^2-a^2}$$

$$= (x)^{\cancel{a^2}-\cancel{b^2}+\cancel{b^2}-\cancel{c^2}+\cancel{c^2}-\cancel{a^2}}$$

$$= x^{0} = 1 = R.H.S$$

Hence proved.

(3) Simplify

(i)

$$\frac{2^{1/3}\times(27)^{1/3}\times(60)^{1/2}}{(180)^{1/2}\times(4)^{-1/3}\times(9)^{1/4}}$$

$$=\frac{2^{1/3}\times 3^{3\times 1/3}\times(2\times2\times3\times5)^{1/2}}{(2\times2\times3\times3\times5)^{1/2}\times(2^2)^{-1/3}\times(3^2)^{1/4}}$$

$$=\frac{2^{1/3}\times 3^{1}\times 2^{2\times 1/2}\times 3^{1/2}\times 5^{1/2}}{2^{2\times 1/2}\times 3^{2\times 1/2}\times 5^{1/2}\times(2)^{-2/3}\times(3)^{1/2}}$$

$$=\frac{2^{1/3}\times\cancel{3^{1}}\times\cancel{2^{1}}\times\cancel{3^{1/2}}\times\cancel{5^{1/2}}}{\cancel{2^{1}}\times\cancel{3^{1}}\times\cancel{5^{1/2}}\times 2^{-2/3}\times\cancel{3^{1/2}}}$$

$$=2^{1/3+2/3}=2^{3/3}=\boxed{2^1}$$

$$=\boxed{2}$$

(ii)

$$\sqrt{\frac{(216)^{2/3}\times(25)^{1/2}}{(0.04)^{-1/2}}}$$

$$=\sqrt{\frac{(2^3\times3^3)^{2/3}\times(5^2)^{1/2}}{\left(\frac{\cancel{4}}{\cancel{100}\ 25}\right)^{-1/2}}}$$

$$=\sqrt{\frac{2^{3\times 2/3}\times 3^{3\times 2/3}\times 5^{1}}{\left(\frac{1}{25}\right)^{-1/2}}}$$

$$=\sqrt{\frac{2^2\times3^2\times5^1}{\frac{1}{5^{2\times -1/2}}}}$$

$$=\sqrt{\frac{2^2\times3^2\times5^1}{\frac{1}{5^{-1}}}}$$

$$=\sqrt{\frac{2^2\times3^2\times\cancel{5^1}}{\cancel{5^1}}}$$

$$=\sqrt{2^{\cancel{2}}}\times\sqrt{3^{\cancel{2}}}$$

$$=2\times3=\boxed{6}$$

R.W

2	216
2	108
2	54
3	27
3	9
3	3
	1

(iii)

$$5^{2^3}\div(5^2)^3$$

$$=5^8\div5^6$$

$$=5^{8-6}=5^2$$

$$=\boxed{25}$$

$2^3=8$

$2\times2\times2=8$

(iv) (3)

$(x^3)^2 \div x^{3^2}$

$x^6 \div x^9$

$= x^{6-9}$

$= \frac{x^{-3}}{1}$

$= \boxed{\frac{1}{x^3}}$

R-W

$3^2 = 3 \times 3$

$= 9$

Complex Number: A number of the form $z = a + bi$ where a and b are real numbers and $i = \sqrt{-1}$ is called a Complex number and represented by z i.e $\boxed{z = a + ib}$

Integral Powers of i

$\boxed{i = \sqrt{-1}}$

$\boxed{(i)^2 = (\sqrt{-1})^2 = -1}$

$(i)^3 = (i)^2 \cdot i$

$\boxed{i^3 = (-1)i = (-i)}$

$(i)^4 = (i)^2 \times (i)^2$

$\boxed{(i)^4 = -1 \times -1 = 1}$.

$i^5 = i^4 \cdot i = (1) i$

$\boxed{i^5 = i}$

Exercise 2.5

(1) ~~Evalutad~~ Evaluate.

$i^7 = (i^2)^3 \cdot i$

$= (-1)^3 i$

$= (-1) i = \boxed{-i}$

ii)

$i^{50} = (i^2)^{25}$

$= (-1)^{25} = \boxed{-1}$

iii) $i^{12} = (i^2)^6$

$= (-1)^6 = \boxed{+1}$

Note: If Power of −ve number is even then Ans is +ve and if Power of −ve number is odd then Ans is −ve

iv)

$(-i)^8 = i^8$

$= (i^2)^4$

$= (-1)^4 = \boxed{+1}$

v) $(-i)^5 = (-i)^4(-i)$

$= (i)^4(-i)$

$= (+1)(-i)$

$(-i)^5 = -i$

vi) $(i^{27}) = i^{26} \cdot i$

$= (i^2)^{13} \cdot i$

$= (-1)^{13} \cdot i$

$= (-1) i = \boxed{-i}$

Exercise 2.5

Conjugate of a complex number:

If we change i to $-i$ in $z = a+bi$, we obtain another complex number $\bar{z} = a-bi$ called the complex conjugate of z. ($\bar{z}$ = read z bar)

Note: In $z = a+bi$
a is called a real part and
b is called the imaginary part
So for conjugate we change only the sign of imaginary part not real number part "a".

e.g. $z = -1-i$
$\bar{z} = -1+i$

(2) Write the conjugate of the following numbers

i) $z = 2+3i$
$\bar{z} = 2-3i$

ii) $z = 3-5i$
$\bar{z} = 3+5i$

iii) $z = -i$
$\bar{z} = i$

iv) $z = -3+4i$
$\bar{z} = -3-4i$

v $z = -4-i$
$\bar{z} = -4+i$

vi $z = i-3$
$\bar{z} = -i+3$

(3) Write the real and imaginary part of the following numbers

i) $z = 1+i$
$Re(z) = 1$ and $Im(z) = 1$

ii) $z = -1+2i$
$Re(z) = -1$ and $Im(z) = 2$

iii) $z = -3i+2$
$Re(z) = 2$ and $Im(z) = -3$

iv) $z = -2-2i$
$Re(z) = -2$ $Im(z) = -2$

v) $z = -3i+0$
$Re(z) = 0$ and $Im(z) = -3$

vi) $z = 2+0i$
$Re\, z = 2$ and $Im(z) = 0$

(4) Find the value of x and y

$x + iy + 1 = 4 - 3i$
$x+1 + iy = 4 - 3i$

Compare the Real and Imaginary Parts

Real Part	imaginary part
$x+1 = 4$	$\boxed{y = -3}$
$x = 4-1$	
$\boxed{x = 3}$	

Exercise 2.6

Q1 Identify the following statements are true OR false

i) $\sqrt{-3}\sqrt{-3}=3$ $\boxed{F}$

because $(\sqrt{-3})(\sqrt{-3})=(\sqrt{-3})^2=-3$ Ans

ii) $i^{73}=-i$ $\boxed{F}$

because $i^{73}=i^{72}\cdot i$

$=(i^2)^{36}\cdot i=(-1)^{36}i=(1)i=(i)$

iii) Complex Conjugate of $(-6i+i^2)$ is $(-1+6i)$ $\boxed{T}$

because $(-6i+i^2)=(-6i+(-1))=-6i-1=-1+6i$

v) Difference of a complex number $z=a+bi$ and its conjugate is a real number $\boxed{F}$

because $z=a+bi$ and $\bar{z}=a-bi$

and diff $=z-\bar{z}=(a+bi)-(a-bi)=\cancel{a}+bi-\cancel{a}+bi$

$z-\bar{z}=2bi$ (Imaginary number)

iii) $i^{10}=-1$ $\boxed{T}$

because $i^{10}=(i^2)^5=(-1)^5=-1$

vi) If $(a-1)-(b+3)i=5+8i$ Then $a=6$ and $b=-11$ $\boxed{T}$

because $a-1=5\Rightarrow\boxed{a=6}$ and $-(b+3)=8\Rightarrow-b-3=8$

$-3-8=b\Rightarrow\boxed{b=-11}$

vii) Product of a complex number and its conjugate is always a non-negative real number. $\boxed{T}$

because if $z=2+i$ and $\bar{z}=2-i$ then $z\bar{z}=(2+i)(2-i)$

$=2^2-(i)^2$

$=4-(-1)$

$=4+1=\boxed{5}$ realnumber

Q2 Express each Complex number in the Standard form $a+bi$ where a & b are real numbers.

i) $2+3i+7-2i$

$= 2+7+3i-2i$

$= \boxed{9+i}$

ii) $2(5+4i)-3(7+4i)$

$= 10+8i-21-12i$

$= 10-21+8i-12i$

$= \boxed{-11-4i}$

iii) $-(-3+5i)-(4+9i)$

$= +3-5i-4-9i$

$= 3-4-5i-9i$

$= \boxed{-1-14i}$

iv) $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$

$= 2(-1)+6(-i)+3(i^2)^8-6(i^2)^9 i+4i$

$= -2-6i+3(-1)^8-6(-1)^9 i+4i$

$= -2-6i+3-6(-1)i+4i$

$= -2-\cancel{6i}+3+\cancel{6i}+4i$

$= -2+3+4i$

$= \boxed{1+4i}$

Q2 Simplify and write your Answer in the form of $a+bi$

i) $(-7+3i)(-3+2i)$

$= 21-14i-9i+6i^2$

$= 21-23i+6(-1)$

$= 21-23i-6$

$= 21-6-23i$

$= \boxed{15-23i}$

ii) $(2-\sqrt{-4})(3-\sqrt{-4})$

R.W

$\sqrt{-4}=2i$

$i^2=-1$

$= (2-2i)(3-2i)$

$= 2(3-2i)-2i(3-2i)$

$= 6-4i-6i+4i^2$

$= 6-10i-4$

$= 6-4-10i$

$= \boxed{2-10i}$

iii) $(\sqrt{5}-3i)^2$

$= (\sqrt{5})^2-2(\sqrt{5})(3i)+(3i)^2$

$= 5-6\sqrt{5}i+9i^2$

$= 5-6\sqrt{5}i-9$

$= \boxed{-4-6\sqrt{5}i}$

iv) $(2-3i)\overline{(3-2i)}$

$= (2-3i)(3+2i)$

$= 2(3+2i)-3i(3+2i)$

$= 6+4i-9i-6i^2$

$= 6-5i-6(-1)$

$= 6-5i+6$

$= \boxed{12-5i}$

(4) Simplify and write your Answer in the form $a+bi$

(i)

$$\frac{-2}{1+i} \times \frac{1-i}{1-i}$$

We Know

$(a-b)(a+b) = a^2-b^2$

$i^2=-1$

$$= \frac{-2(1-i)}{(1+i)(1-i)}$$

$$= \frac{-2+2i}{1^2-i^2}$$

$$= \frac{-2+2i}{1-(-1)}$$

$$= \frac{+2(-1+i)}{1+1}$$

$$= \frac{\cancel{2}(-1+i)}{\cancel{2}} = \boxed{-1+i}$$

ii)

$$\frac{2+3i}{4-i}$$

Multiply and Divide by Conjugate of $4+i$

$$\frac{2+3i}{4-i} \times \frac{4+i}{4+i}$$

$$= \frac{(2+3i)(4+i)}{(4-i)(4+i)}$$

$$= \frac{2(4+i)+3i(4+i)}{4^2-i^2}$$

$$= \frac{8+2i+12i+3i^2}{16-(-1)}$$

$$= \frac{8+14i+3(-1)}{16+1}$$

$$= \frac{8-3+14i}{17}$$

$$= \frac{5+14i}{17}$$

$$= \frac{5}{17} + \frac{14}{17}i$$

iii)

$$\frac{9-7i}{3+i}$$

$$\frac{9-7i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{(9-7i)(3-i)}{(3+i)(3-i)}$$

$$= \frac{9(3-i)-7i(3-i)}{3^2-i^2}$$

$$= \frac{27-9i-21i+7i^2}{9-(-1)}$$

$$= \frac{27-30i+7(-1)}{9+1}$$

$$= \frac{27-7-30i}{10}$$

$$= \frac{20-30i}{10}$$

$$= \frac{\cancel{10}(2-3i)}{\cancel{10}}$$

$$= 2-3i$$

(4) iv)

$$\frac{2-6i}{3+i} - \frac{4+i}{3+i}$$

$$= \frac{(2-6i)-(4+i)}{(3+i)}$$

$$= \frac{2-6i-4-i}{(3+i)}$$

$$= \frac{-2-7i}{3+i}$$

Now multiply and Divide by $3-i$

$$= \frac{-2-7i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{(-2-7i)(3-i)}{(3+i)(3-i)}$$

$$= \frac{-2(3-i)-7i(3-i)}{3^2-i^2}$$

$$= \frac{-6+2i-21i+7i^2}{9-(-1)}$$

$$= \frac{-6-19i+7(-1)}{9+1}$$

$$= \frac{-6-7-19i}{10}$$

$$= \frac{-13-19i}{10}$$

$$= \boxed{\frac{-13}{10} - \frac{19}{10}i}$$

(V)

$$\left(\frac{1+i}{1-i}\right)^2 = \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^2$$

$$= \left[\frac{(1+i)(1+i)}{(1-i)(1+i)}\right]^2$$

$$= \left[\frac{1(1+i)+i(1+i)}{1^2-i^2}\right]^2$$

$$= \left[\frac{1+i+i+i^2}{1-(-1)^2}\right]^2$$

$$= \left[\frac{\cancel{1}+2i-\cancel{1}}{1+1}\right]^2 = \left[\frac{\cancel{2}i}{\cancel{2}}\right]^2$$

$$= [i]^2 = -1$$

$$= \boxed{-1+0i}$$

vi)

$$\frac{1}{(2+3i)(1-i)} = \frac{1}{2(1-i)+3i(1-i)}$$

$$= \frac{1}{2-2i+3i-3i^2}$$

$$= \frac{1}{2+i-3(-1)} = \frac{1}{2+i+3}$$

$$= \frac{1}{5+i}$$

Now multiply and Divide by $(5-i)$

$$= \frac{1}{(5+i)} \times \frac{5-i}{(5-i)}$$

$$= \frac{5-i}{5^2-i^2}$$

$$= \frac{5-i}{25-(-1)} = \frac{5-i}{25+1}$$

$$= \frac{5-i}{26} = \boxed{\frac{5}{26} - \frac{i}{26}}$$

(5) Calculate (a) $\bar{z}$ (b) $z+\bar{z}$
(c) $z-\bar{z}$ (d) $z\bar{z}$

i) $z = -i$

(a) $\bar{z} = i$ (conjugate of z)

b) $z+\bar{z} = -i+i = 0$

c) $z-\bar{z} = -i-i = -2i$

d) $z\bar{z} = (-i)(i)$
$= -i^2$
$z\bar{z} = -(-1) = +1$

ii) $z = 2+i$

(a) $\boxed{\bar{z} = 2-i}$

(b) $z+\bar{z} = 2+\cancel{i}+2-\cancel{i}$
$\boxed{z+\bar{z} = 4}$

(c) $z-\bar{z} = (2+i)-(2-i)$
$= \cancel{2}+i-\cancel{2}+i$
$\boxed{z-\bar{z} = 2i}$

d) $z\bar{z} = (2+i)(2-i)$
$= 2^2 - i^2$
$= 4-(-1) = 4+1$
$\boxed{z\bar{z} = 5}$

iii) $z = \frac{1+i}{1-i}$

First change z in the form of a+bi by multiply and Dividing by 1+i

$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$= \frac{(1+i)(1+i)}{(1-i)(1+i)}$

$= \frac{1(1+i)+i(1+i)}{1^2-i^2}$

$= 1+i+i+i^2$

$= \frac{1+2i+(-1)}{1-(-1)}$

$= \frac{\cancel{1}+2i-\cancel{1}}{1+1}$

$z = \frac{\cancel{2}i}{\cancel{2}} = i$

(a) $\bar{z} = -i$

(b) $z+\bar{z} = \cancel{i}-\cancel{i} = 0$

c) $z-\bar{z} = i-(-i)$.
$z-\bar{z} = i+i = 2i$

d) $z\bar{z} = (i)(-i)$
$= -i^2 = -(-1)$
$\boxed{z\bar{z} = 1}$

iv) $z = \frac{4-3i}{2+4i}$

First change z in the form a+bi by multiply and Dividing by 2-4i

$z = \frac{4-3i}{2+4i} \times \frac{2+4i}{2-4i}$

See on Next Page-

(5) iv)

$$Z = \frac{(4-3i)(2-4i)}{(2+4i)(2-4i)}$$

$$= \frac{4(2-4i) - 3i(2-4i)}{2^2 - (4i)^2}$$

$$= \frac{8 - 16i - 6i + 12i^2}{4 - 16i^2}$$

$$= \frac{8 - 22i + 12(-1)}{4 - 16(-1)}$$

$$= \frac{8 - 22i - 12}{4 + 16}$$

$$= \frac{-4 - 22i}{20}$$

$$= \frac{\cancel{2}(-2 - 11i)}{\cancel{20}_{10}}$$

$$Z = \frac{-\cancel{2}^{1}}{\cancel{10}_{5}} - \frac{11i}{10}$$

$$Z = -\frac{1}{5} - \frac{11i}{10}$$

(a) $\boxed{\bar{Z} = -\frac{1}{5} + \frac{11i}{10}}$

(b) $Z + \bar{Z} = -\frac{1}{5} - \cancel{\frac{11i}{10}} - \frac{1}{5} + \cancel{\frac{11i}{10}}$

$$= \boxed{\frac{-2}{5}}$$

(c) $Z - \bar{Z} = \left(-\frac{1}{5} - \frac{11i}{10}\right) - \left(-\frac{1}{5} + \frac{11i}{10}\right)$

$$= -\cancel{\frac{1}{5}} - \frac{11i}{10} + \cancel{\frac{1}{5}} - \frac{11i}{10}$$

$$Z - \bar{Z} = \frac{-22i}{10} = \boxed{\frac{-11i}{5}}$$

(d) $Z\bar{Z} = \left(-\frac{1}{5} - \frac{11i}{10}\right)\left(-\frac{1}{5} + \frac{11i}{10}\right)$

$$= \left(-\frac{1}{5}\right)^2 - \left(\frac{11i}{10}\right)^2$$

$$= \frac{1}{25} - \left(\frac{121}{100} i^2\right)$$

$$= \frac{1}{25} - \frac{121(-1)}{100}$$

$$= \frac{1 \times 4}{25 \times 4} + \frac{121}{100}$$

$$= \frac{4 + 121}{100} = \frac{\cancel{125}^{25}}{\cancel{100}_{20}}$$

$$= \frac{\cancel{25}^{5}}{\cancel{20}_{4}} = \boxed{\frac{5}{4}}$$

(6) If $Z = 2+3i$ and $W = 5-4i$ then show that

(a) $\overline{Z+W} = \bar{Z} + \bar{W}$

L.H.S $= \overline{Z+W}$

first find $Z + W = 2+3i+5-4i$

$$= 7 - i$$

L.H.S $= \overline{Z+W} = \overline{(7-i)}$

$$= 7 + i \quad \text{— (1)}$$

For R.H.S $= \bar{Z} = 2 - 3i$

$$\bar{W} = 5 + 4i$$

R.H.S $= \bar{Z} + \bar{W}$

$$= 2 - 3i + 5 + 4i$$

$$= 7 + i \quad \text{— (2)}$$

From (1) and (2) we have proved

$\overline{Z+W} = \bar{Z} + \bar{W}$

(6) (ii) If $z = 2+3i$ and $w = 5-4i$ then Show that $\overline{z-w} = \bar{z} - \bar{w}$

Sol: $z - w = (2+3i) - (5-4i)$

$= 2+3i-5+4i$

$z - w = -3+4i$

$L.H.S = \overline{z-w} = -3-4i$ — (1)

$R.H.S = \bar{z} - \bar{w}$

$= (2-3i) - (5+4i)$

$= 2-3i-5-4i$

$R.H.S = -3-4i$ — (2)

from (1) and (2) we have Proved That $L.H.S = R.H.S$

(iii) $\overline{zw} = \bar{z}\,\bar{w}$

For L.H.S First find zw

$zw = (2+3i)(5-4i)$

$= 2(5-4i)+3i(5-4i)$

$= 10-8i+15i-12i^2$

$= 10+7i-12(-1)$

$= 10+7i+12$

$zw = 22+7i$

$L.H.S = \overline{zw} = \overline{22+7i}$

$= 22-7i$ — (1)

For R.H.S First find $\bar{z}$ and $\bar{w}$ then multiply

$\bar{z} = 2-3i$

$\bar{w} = 5+4i$

$R.H.S = \bar{z}\,\bar{w}$

$= (2-3i)(5+4i)$

$= 2(5+4i)-3i(5+4i)$

$= 10+8i-15i-12i^2$

$= 10-7i-12(-1)$

$= 10-7i+12$

$\bar{z}\,\bar{w} = 22-7i$ — (2)

from (1) and (2) we have Proved that $L.H.S = R.H.S.$

(iv) $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$

For L.H.S First find $\frac{z}{w}$ then find $\overline{\left(\frac{z}{w}\right)}$

$\frac{z}{w} = \frac{2+3i}{(5-4i)}$

$\frac{z}{w} = \frac{(2+3i)(5+4i)}{(5-4i)(5+4i)}$

$= \frac{2(5+4i)+3i(5+4i)}{5^2-(4i)^2}$

$= \frac{10+8i+15i+12i^2}{25-16(i^2)}$

$= \frac{10+23i+12(-1)}{25-16(-1)}$

$= \frac{10+23i-12}{25+16}$

$\frac{z}{w} = \frac{-2+23i}{41} =$

(Q iv) Remaing Part

$$\frac{z}{w} = \frac{-2}{41} + \frac{23i}{41}$$

$$\left(\overline{\frac{z}{w}}\right) = \frac{-2}{41} - \frac{23i}{41} \quad — (1)$$

For R.H.S First find $\bar{z}$ and $\bar{w}$ then find $\frac{\bar{z}}{\bar{w}}$

$\bar{z} = 2-3i$
$\bar{w} = 5+4i$

$$\frac{\bar{z}}{\bar{w}} = \frac{(2-3i)}{(5+4i)}$$

$$R.H.S = \frac{\bar{z}}{\bar{w}} = \frac{(2-3i)(5-4i)}{(5+4i)(5-4i)}$$

$$= \frac{2(5-4i)-3i(5-4i)}{5^2-(4i)^2}$$

$$= \frac{10-8i-15i+12i^2}{25-16(i^2)}$$

$$= \frac{10-23i+12(-1)}{25-16(-1)}$$

$$= \frac{10-23i-12}{25+16}$$

$$R.H.S = \frac{-2-23i}{41}$$

$$= \frac{-2}{41} - \frac{23}{41}i \quad — (2)$$

From (1) and (2) we have proved
L.H.S = R.H.S

V) If $z = 2+3i$ and $w = 5-4i$ then show that $\frac{1}{2}(z+\bar{z})$ is the real part

$$\frac{1}{2}(z+\bar{z})$$

$$= \frac{1}{2}(2+\cancel{3i}+2-\cancel{3i})$$

$$= \frac{1}{\cancel{2}}(4)^2$$

$$= \boxed{2} \quad \text{real Part}$$

(vi) $\frac{1}{2i}(z-\bar{z})$

$$= \frac{1}{2i}\left[(2+3i)-(2-3i)\right]$$

$$= \frac{1}{2i}\left(\cancel{2}+3i-\cancel{2}+3i\right]$$

$$= \frac{1}{\cancel{2i}}\left(\overset{3}{\cancel{6i}}\right)$$

$= 3$ real Part (wrong value of Question paper)

Correct value is

vi) $\frac{1}{2}(z-\bar{z})$

$$= \frac{1}{2}\left[(2+3i)-(2-3i)\right]$$

$$= \frac{1}{2}\left[\cancel{2}+3i-\cancel{2}+3i\right]$$

$$= \frac{1}{\cancel{2}}\left[\overset{3}{\cancel{6}}i\right]$$

$= 3i$ imaginary Parts

hence Prove.

(1) Solve the following Questions for real x and y

(i) $(2-3i)(x+yi) = 4+i$

$2(x+yi) - 3i(x+yi) = 4+i$

$2x + 2yi - 3ix - 3yi^2 = 4+i$

$2x + 2yi - 3ix - 3y(-1) = 4+i$

$2x + 2yi - 3ix + 3y = 4+i$

$(2x+3y) + (-3x+2y)i = 4+i$

Compare the Real Parts and Imaginary Part on both sides

$2x+3y = 4 \quad -(1)$ | $-3x+2y = 1 \quad -(2)$

Multiply Eq (1) by 3 and Eq (2) by 2 then add them

$$\cancel{6x} + 9y = 12$$
$$-\cancel{6x} + 4y = 2$$
$$13y = 14$$
$$\boxed{y = 14/13}$$

Now put This value in Eq (1) we have

$2x + 3(14/13) = 4$

$\frac{2x}{1} + \frac{42}{13} = \frac{4}{1}$

For removing 13 from denominator multiply by 13 (LCM)

$13(2x) + \frac{42}{\cancel{13}} \times \cancel{13} = 4 \times 13$

$26x + 42 = 52$

$26x = 52 - 42$

$26x = 10$

$$\boxed{x = \frac{10}{26} = \frac{5}{13}}$$

(ii) $(3-2i)(x+yi) = 2(x-2yi) + 2i - 1$

$3(x+yi) - 2i(x+yi) = 2x - 4yi + 2i - 1$

$3x + 3yi - 2xi - 2yi^2 = 2x - 4yi + 2i - 1$

$3x - 2x + 3yi + 4yi - 2xi - 2y(-1) = 2i - 1$

$x + 7yi - 2xi + 2y = 2i - 1$

$(x+2y) + (-2x+7y)i = -1 + 2i$

Now Compare The real Part and Imaginary Parts.

$x + 2y = -1 \quad$ — I

$-2x + 7y = 2 \quad$ — II

Multiply Eq I by 2 and add in II

$$2\cancel{x} + 4y = -2$$
$$-\cancel{2}x + 7y = 2$$
$$11y = 0$$
$$\boxed{y = 0/11 = 0}$$

Now for value of x Put $y=0$ in Eq I we have

$x + 2(0) = -1$

$x + 0 = -1$

$$\boxed{x = -1}$$

2 (iii)

$$(3+4i)^2 - 2(x-yi) = x+yi$$

$$9+16i^2+24i-2x+2yi = x+yi$$

$$9+16(-1)+24i-2x-x+2yi-yi=0$$

$$9-16+24i-3x+yi=0$$

$$-7+24i-3x+yi=0$$

$$-3x+yi=7-24i$$

Compare the Real and Imaginary Parts

$-3x=7$ | $y=-24$

$$\boxed{x=\frac{7}{3}}$$

Review Exercise 2 - P#54

3/ Simplify

i) $\sqrt[4]{81y^{-12}x^{-8}}$

$$=(81)^{1/4}(y^{-12})^{1/4}(x^{-8})^{1/4}$$

$$=(3^4)^{\times\frac{1}{4}}(y)^{-12\times\frac{1}{4}}x^{-8\times\frac{1}{4}}$$

$$=3^1y^{-3}x^{-2}$$

$$=\boxed{\frac{3}{y^3x^2}}$$

ii) $\sqrt{25x^{10n}y^{8m}}$

$$=(25x^{10n}y^{8m})^{1/2}$$

$$=(25)^{1/2}(x^{10n})^{1/2}(y^{8m})^{1/2}$$

$$=5^{2\times\frac{1}{2}}x^{10n\times\frac{1}{2}}y^{8m\times\frac{1}{2}}$$

$$=\boxed{5x^{5n}y^{4m}}$$

iii) $\left(\frac{x^3y^4z^5}{x^{-2}y^{-1}z^{-5}}\right)^{1/5}$

$$=\left(x^{3+2}y^{4+1}z^{5+5}\right)^{1/5}$$

$$=\left(x^5y^5z^{10}\right)^{1/5}$$

$$=x^{5\times\frac{1}{5}}y^{5\times\frac{1}{5}}z^{10\times\frac{1}{5}}$$

$$=\boxed{xyz^2}$$

iv) $\left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}}\right)^{2/5}=$

$32=2^5$

$625=5^4$

$$=\left(\frac{2^5}{5^4}x^{-6-4}y^{-4-1}z^{1+4}\right)^{2/5}$$

$$=\left(\frac{2^{5\times\frac{2}{5}}x^{-10\times\frac{2}{5}}y^{-5\times\frac{2}{5}}z^{5\times\frac{2}{5}}}{5^{4\times 2/5}}\right)$$

$$=\left(\frac{2^2x^{-4}y^{-2}z^2}{5^{8/5}}\right)=\boxed{\frac{4z^2}{5^{8/5}x^4y^2}}$$

(4) Simplify $\sqrt{\dfrac{(216)^{2/3} \times (25)^{1/2}}{(0.04)^{-3/2}}}$

$= \sqrt{\dfrac{(6^3)^{2/3} \times (5^2)^{1/2}}{\left(\frac{4}{100}\right)^{-3/2}}}$

$= \sqrt{\dfrac{6^2 \times 5}{\left(\frac{1}{25}\right)^{-3/2}}}$

$= \sqrt{\dfrac{6^2 \times 5}{\frac{1}{5^{2\times -3/2}}}} = \sqrt{\dfrac{6^2 \times 5}{\frac{1}{5^{-3}}}}$

$= \sqrt{\dfrac{6^2 \times 5^1}{5^3}} = \sqrt{\dfrac{6^2}{5^{3-1}}}$

$= \sqrt{\dfrac{6^2}{5^2}} = \dfrac{6^{2\times\frac{1}{2}}}{5^{2\times\frac{1}{2}}} = \boxed{\dfrac{6}{5}}$

(5) Simplify

$\left(\dfrac{a^p}{a^q}\right)^{p+q} \cdot \left(\dfrac{a^q}{a^r}\right)^{q+r} \div 5(a^p \cdot a^r)^{p-r}$

$= (a^{p-q})^{p+q} \cdot (a^{q-r})^{q+r} \div 5(a^{p+r})^{p-r}$

$= \dfrac{(a^{p^2-q^2})(a^{q^2-r^2})}{5(a^{p^2-r^2})}$

$= \dfrac{a^{p^2-q^2+q^2-r^2-p^2+r^2}}{5} = \dfrac{a^0}{5}$

$= \boxed{\dfrac{1}{5}}$

(6)

$\left(\dfrac{a^{2l}}{a^{l+m}}\right)\left(\dfrac{a^{2m}}{a^{m+n}}\right)\left(\dfrac{a^{2n}}{a^{n+l}}\right)$

$= \left(a^{2l-l-m}\right)\left(a^{2m-m-n}\right)\left(a^{2n-n-l}\right)$

$= \left(a^{l-m}\right)\left(a^{m-n}\right)\left(a^{n-l}\right)$

$= a^{\cancel{l}-\cancel{m}+\cancel{m}-\cancel{n}+\cancel{n}-\cancel{l}} = a^0$

$= \boxed{1}$

(7)

$\sqrt[3]{\dfrac{a^l}{a^m}} \times \sqrt[3]{\dfrac{a^m}{a^n}} \times \sqrt[3]{\dfrac{a^n}{a^l}}$

$= (a^{l-m})^{1/3} \times (a^{m-n})^{1/3} \times (a^{n-l})^{1/3}$

$= \left(a^{\frac{1}{3}l-\frac{1}{3}m}\right) \times (a)^{\frac{1}{3}m-\frac{1}{3}n} \times (a)^{\frac{1}{3}n-\frac{1}{3}l}$

$= a^{\frac{1}{3}\cancel{l}-\frac{1}{3}\cancel{m}+\frac{1}{3}\cancel{m}-\frac{1}{3}\cancel{n}+\frac{1}{3}\cancel{n}-\frac{1}{3}\cancel{l}}$

$= a^0 = \boxed{1}$

Note: In the radical form $\boxed{\sqrt[n]{x}}$

$\sqrt{\ }$ is called radical sign

x is called radicand or base

n is called index of radical.

Some examples of radical and exponential form

Radical form	exponential form
$\sqrt{x}$	$x^{1/2}$ (Square root)
$\sqrt[3]{x}$	$x^{1/3}$ (cube root)
$\sqrt[4]{x}$	$x^{1/4}$ (fourth root)
$\vdots$	$\vdots$
$\sqrt[n]{x}$	$x^{\frac{1}{n}}$ (nth root)