Subject: Maths (Ex # 5.1) Chapter # 5 Azizullah Class IX

FFC Model School

Factorize

Q1 (i) $2abc - 4abx + 2abd$

$2ab(c - 2x + d)$

(ii) $9xy - 12x^2y + 18y^2$

$3y(3x - 4x^2 + 6y)$

(iii) $-3x^2y - 3x + 9xy^2$

$-3x(xy + 1 - 3y^2)$

(iv) $5ab^2c^3 - 10a^2b^3c - 20a^3bc^2$

$5abc(bc^2 - 2ab^2 - 4a^2c)$

(v) $3x^3y(x-3y) - 7x^2y^2(x-3y)$

$(3x^3y - 7x^2y^2)(x-3y)$

$x^2y(3x - 7y)(x-3y)$

(vi) $2xy^3(x^2+5) + 8xy^2(x^2+5)$

$= (2xy^3 + 8xy^2)(x^2+5)$

$= 2xy^2(y+4)(x^2+5)$

Q2 $5ax - 3ay - 5bx + 3by$

$= a(5x-3y) - b(5x-3y)$

$= (5x-3y)(a-b)$

2 (ii) $3xy + 2y - 12x - 8$

$= y(3x+2) - 4(3x-2)$

$= (3x+2)(y-4)$

(iii) $x^3 + 3xy^2 - 2x^2y - 6y^3$

$= x(x^2+3y^2) - 2y(x^2+3y)$

$= (x-2y)(x^2+3y^2)$

(iv) $(x^2-y^2)z + (y^2-z^2)x$

$= x^2z - y^2z + y^2x - z^2x$

$= y^2x - y^2z + x^2z - z^2x$

$= y^2(x-z) + xz(x-z)$

$= (x-z)(y^2+xz)$

Q3 (i) $144a^2 + 24a + 1$

$= (12a)^2 + 2(12a)(1) + (1)^2$

$= (12a+1)^2$

(ii) $\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}$

$= \left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2$

$= \left(\frac{a}{b} - \frac{b}{a}\right)^2$

Exercise 5-1

Q3 iii) $(x+y)^2 - 14z(x+y) + 49z^2$

$= (x+y)^2 - 2(7z)(x+y) + (7z)^2$

$= (x+y-7z)^2$

iv) $12x^2 - 36x + 27$

$= 3(4x^2 - 12x + 9)$

$= 3\left[(2x)^2 - 2(2x)(3) + (3)^2\right]$

$= 3\left[(2x-3)^2\right]$

Q4 (i) $3x^2 - 75y^2$

$= 3(x^2 - 25y^2)$

$= 3\left[(x)^2 - (5y)^2\right]$

$= 3(x+5y)(x-5y)$

(ii) $x(x-1) - y(y-1)$

$= x^2 - x - y^2 + y$

$= x^2 - y^2 - x + y$

$= (x+y)(x-y) - 1(x-y)$

$= (x-y)(x+y-1)$

(iii) $128am^2 - 242an^2$

$= 2a(64m^2 - 121n^2)$

$= 2a\left[(8m)^2 - (11n)^2\right]$

$2a(8m+11n)(8m-11n)$

iv) $3x - 243x^3$

$= 3x(1 - 81x^2)$

$= 3x\left[(1)^2 - (9x)^2\right]$

$= 3x(1+9x)(1-9x)$

Q5 (i) $x^2 - y^2 - 6y - 9$

$= x^2 - (y^2 + 6y + 9)$

$= x^2 - \left[(y)^2 + 2(y)(3) + (3)^2\right]$

$= x^2 - (y+3)^2$

$= (x+y+3)\left[x - (y+3)\right]$

$= (x+y+3)(x-y-3)$

ii) $x^2 - a^2 + 2a - 1$

$= x^2 - \left[a^2 - 2a + 1\right]$

$= x^2 - \left[(a)^2 - 2(a)(1) + (1)^2\right]$

$= x^2 - \left[(a-1)^2\right]$

$= \left[x + (a-1)\right]\left[x - (a-1)\right]$

$= (x+a-1)(x-a+1)$

iii) $4x^2 - y^2 - 2y - 1$

$= 4x^2 - (y^2 + 2y + 1)$

$= (2x)^2 - (y+1)^2$

$= \left[2x + (y+1)\right]\left[2x - (y+1)\right]$

$= (2x+y+1)(2x-y-1)$

Q 5 (iv)

$$x^2 - y^2 - 4x - 2y + 3$$
$$= x^2 - y^2 - 4x - 2y + 3 + 1 - 1$$
$$= \underline{x^2 - 4x + 4} - \underline{y^2 - 2y - 1}$$
$$= (x-2)^2 - (y^2 + 2y + 1)$$
$$= (x-2)^2 - (y+1)^2$$
$$= [(x-2) + (y+1)][(x-2) - (y+1)]$$
$$= (x - 2 + y + 1)(x - 2 - y - 1)$$
$$= (x + y - 1)(x - y - 3)$$

Q 5 (v) $25x^2 - 10x + 1 - 36z^2$

$$= (5x)^2 - 2(5x)(1) + (1)^2 - (6z)^2$$
$$= (5x - 1)^2 - (6z)^2$$
$$= (5x - 1 - 6z)(5x - 1 + 6z)$$

(vi) $x^2 - y^2 - 4xz + 4z^2$

$$= x^2 + 4z^2 - 4xz - y^2$$
$$= (x)^2 + (2z)^2 - 2(x)(2z) - (y)^2$$
$$= (x - 2z)^2 - (y)^2$$
$$= (x - 2z + y)(x - 2z - y)$$
$$= (x + y - 2z)(x - y - 2z)$$

Factorize

1/ (i) $x^4 + \frac{1}{x^4} - 3$

$$= x^4 + \frac{1}{x^4} - 2 - 1$$
$$= (x^2)^2 + \left(\frac{1}{x^2}\right)^2 - 2 - (1)^2$$
$$= \left(x^2 - \frac{1}{x^2}\right)^2 - (1)^2$$
$$= \left(x^2 - \frac{1}{x^2} + 1\right)\left(x^2 - \frac{1}{x^2} - 1\right)$$

(ii)

$$3x^4 + 12y^4$$
$$= 3[x^4 + 4y^4]$$
$$= 3[x^4 + 4y^4 + 4x^2y^2 - 4x^2y^2]$$
$$= 3[(x^2)^2 + (2y^2)^2 + 2(x^2)(2y^2) - (2xy)^2]$$
$$= 3[(x^2 + 2y^2)^2 - (2xy)^2]$$
$$= 3[(x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)]$$

iii) $a^4 + 3a^2b^2 + 4b^4 + a^2b^2 - a^2b^2$

$$= a^4 + 4a^2b^2 + 4b^4 - a^2b^2$$
$$= (a^2)^2 + 2(a^2)(2b^2) + (2b^2)^2 - (ab)^2$$
$$= (a^2 + 2b^2)^2 - (ab)^2$$
$$= (a^2 + 2b^2 + ab)(a^2 + 2b^2 - ab)$$

Exercise 5.2

Q1 (iv) $4x^4+81$

$\underbrace{(2x^2)^2+(9)^2+2(2x^2)(9)}-2(2x^2)(9)$

$= (2x^2+9)^2 - 36x^2$

$= (2x^2+9)^2 - (6x)^2$

$= (2x^2+9+6x)(2x^2+9-6x)$

v) x^4+x^2+25

$= x^4+25+x^2$

$= \underbrace{(x^2)^2+(5)^2+2(x^2)(5)} - 10x^2+x^2$

$= (x^2+5)^2 - 9x^2$

$= (x^2+5)^2 - (3x)^2$

$= (x^2+5+3x)(x^2+5-3x)$

vi) x^4+4x^2+16

$\underbrace{(x^2)^2+(4)^2+2(x^2)(4)}-2(x^2)(4)+4x^2$

$= (x^2+4)^2 - 8x^2+4x^2$

$= (x^2+4)^2 - 4x^2$

$= (x^2+4)^2 - (2x)^2$

$= (x^2+4+2x)(x^2+4-2x)$

Q2 (i) $1x^2+14x+48$ (Product)

find Product of coefficient x^2 and Constant number 48

Now we change middle Part $14x$ into two Parts i.e $6x+8x=14x$

Product

$1\times 48 = 48$

$6\times 8 = 48$

Sum

$6+8=14$

$= x^2+8x+6x+48$

$= x(x+8)+6(x+8)$

$= (x+8)(x+6)$

Note (1) when product of Co-efficient of x^2 and Constant Number is +ve then we will find sum for middle term.

(2) when Product of coefficient of x^2 and constant number is −ve then we will find difference of factors for middle term;

Exercise 5.2

Q2 (ii)

$1x^2 - 21x + 108$ — Product

$= x^2 - 12x - 9x + 108$

$= x(x-12) - 9(x-12)$

$= (x-12)(x-9)$

Product: $1 \times 108 = 108$
$12 \times 9 = 108$
Sum: $12 + 9 = 21$

(iii)

$1x^2 - 11x - 42$

$= x^2 - 14x + 3x - 42$

$= x(x-14) + 3(x-14)$

$= (x-14)(x+3)$

Product: $1 \times (-42) = -42$
$14 \times 3 = 42$
diff: $-14 + 3 = -11$

iv)

$1x^2 + x - 132$

$= x^2 + 12x - 11x - 132$

$= x(x+12) - 11(x+12)$

$= (x+12)(x-11)$

Product: $1 \times (-132) = -132$
$12 \times 11 = 132$
diff: $12 - 11 = 1$

Q3 (i)

$4x^2 + 12x + 5$ — Product

$4x^2 + 2x + 10x + 5$

$= 2x(2x+1) + 5(2x+1)$

$= (2x+5)(2x+1)$

Product: $4 \times 5 = 20$
$10 \times 2 = 20$
Sum: $10 + 2 = 12$

ii)

$30x^2 + 7x - 15$

$= 30x^2 + 25x - 18x - 15$

$= 5x(6x+5) - 3(6x+5)$

$= (6x+5)(5x-3)$

Product: $30 \times (-15) = -450$
-ve Product
find diff
$18 \times 25 = 450$
$25 - 18 = 7$

Q3 (iii)

$24x^2 - 65x + 21$ — Product

$= 24x^2 - 56x - 9x + 21$

$= 8x(3x-7) - 3(3x-7)$

$= (3x-7)(8x-3)$

Product: $24 \times 21 = 504$
Sum: $56 \times 9 = 504$
$56 + 9 = 65$

iv)

$5x^2 - 16x - 21$

$= 5x^2 + 5x - 21x - 21$

$= 5x(x+1) - 21(x+1)$

$= (x+1)(5x+1)$

Product: $5 \times (-21) = -105$
diff: $-21 + 5 = -16$

(V)

$4x^2 - 17xy + 4y^2$

$= 4x^2 - 16xy - xy + 4y^2$

$= 4x(x-4y) - y(x-4y)$

$= (4x-y)(x-4y)$

Product: $4 \times 4 = 16$
Sum: $16 \times 1 = 17$

vi)

$3x^2 - 38xy - 13y^2$

$3x^2 - 39xy + xy - 13y^2$

$= 3x(x-13y) + y(x-13y)$

$= (3x+y)(x-13y)$

$(3)(-13) = -39$
diff: $39 \times 1 = 39$
$39 - 1 = 38$

vii)

$5x^2 + 33xy - 14y^2$

$= 5x^2 + 35xy - 2xy - 14y^2$

$= 5x(x+7y) - 2y(x+7y)$

$= (5x-2y)(x+7y)$

Product: $(5)(-14) = -70$
$35 \times 2 = 70$
diff: $+35 - 2 = 33$

C.

Exercise 5.2

Q3 viii)

$$\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4$$

Suppose $\left(5x-\frac{1}{x}\right)=y$

$= y^2+4y+4$

$= (y)^2+2(y)(2)+(2)^2$

$= (y+2)^2$

$= (y+2)(y+2)$

Now Replace value of y

$$\left(5x-\frac{1}{x}+2\right)\left(5x-\frac{1}{x}+2\right)$$

Q4 (i) $(x^2+5x+4)(x^2+5x+6)-3$

Let $\boxed{x^2+5x=y}$

$(y+4)(y+6)-3$

$= y^2+10y+24-3$

$= 1y^2+10y+21$

$= y^2+3y+7y+21$

$= y(y+3)+7(y+3)$

$= (y+7)(y+3)$

Product
$1\times 21=21$
$3\times 7=21$
Sum
$3+7=10$

Now Replace the value of y

$(x^2+5x+7)(x^2+5x+3)$

Q 4(ii)

$(x^2-4x)(x^2-4x-1)-20$

Let $y=x^2-4x$

$=(y)(y-1)-20$

$= 1y^2-y-20$

$= y^2-5y+4y-20$

$= y(y-5)+4(y-5)$

$= (y-5)(y+4)$

Product
$1\times(-20)=-20$
$5\times 4=20$
diff
$-5+4=-1$

Now Replace again the value of y

$= (x^2-4x-5)(x^2-4x+4)$

$= (x^2-5x+x-5)(x-2)^2$

$= [x(x-5)+1(x-5)][(x-2)^2]$

$= (x-5)(x+1)(x-2)^2$

Q4 (iii)

$(x+2)(x+3)(x+4)(x+5)-15$

$(x+2)(x+5)(x+3)(x+4)-15$

$(x^2+7x+10)(x^2+7x+12)-15$

Now Suppose $y=x^2+7x$

$(y+10)(y+12)-15$

$= y^2+22y+120-15$

$= y^2+22y+105$

$= y^2+15y+7y+105$

$= y(y+15)+7(y+15)$

$= (y+15)(y+7)$

product
$1\times 105=105$
$15\times 7=105$

$(x^2+7x+15)(x^2+7x+7)$

Q4 (iv) $(x+4)(x-5)(x+6)(x-7)-504$

$(x^2-x-20)(x^2-x-42)-504$

Suppose $x^2-x=y$

$=(y-20)(y-42)-504$

$=y^2-42y-20y+840-504$

$y^2-62y+336$

$=y^2-56y-6y+336$

$=y(y-56)-6(y-56)$

$=(y-56)(y-6)$

Product
$1\times(+336)=+336$
$56\times6=336$
Sum
$56+6=62$

Now Replace value of y

$(x^2-x-56)(x^2-x-6)$

$=[x^2+7x-8x-56][x^2-3x+2x-6]$

$=[x(x+7)-8(x+7)]\{x(x-3)+2(x-3)\}$

$=[(x+7)(x-8)][(x+2)(x-3)]$

Q4 (v) $(x+1)(x+2)(x+3)(x+6)-3x^2$

Hince $1\times6=6$
$2\times3=6$

$(x+1)(x+6)(x+2)(x+3)-3x^2$

$(x^2+7x+6)(x^2+5x+6)-3x^2$

$(x^2+6+7x)(x^2+6+5x)-3x^2$

Suppose $\boxed{x^2+6=y}$

$(y+7x)(y+5x)-3x^2$

$y^2+12xy+35x^2-3x^2$

$=y^2+12xy+32x^2$

$=y^2+8xy+4xy+32x^2$

$=y(y+8x)+4x(y+8x)$

$=(y+4x)(y+8x)$

Product
$1\times32=32$
$8\times4=32$
Sum
$8+4=12$

Replace the value of y.

$=(x^2+6+4x)(x^2+6+8x)$

$=(x^2+4x+6)(x^2+8x+6)$

Q5 (i) $x^3+48x-12x^2-64$

$=x^3-12x^2+48x-64$

$=x^3-12x(x-4)-(4)^3$

$=(x)^3-3(x)(4)(x-4)-(4)^3$

$=(x-4)^3$

use the formula

$(a-b)^3=a^3-3ab(a-b)-b^3$

(ii) $8x^3+60x^2+150x+125$

$=(2x)^3+30x(2x+5)+(5)^3$

$=(2x)^3+3(2x)(5)(2x+5)+(5)^3$

$=(2x+5)^3$

by using the formula.

$(a+b)^3=a^3+3ab(a+b)+b^3$

Q 5 (iii)

$x^3 - 18x^2 + 108x - 216$

$= (x)^3 - 18x(x-6) - (6)^3$

$= (x)^3 - 3(x)(6)(x-6) - (6)^3$

$= (x-6)^3$

iv)

$8x^3 - 125y^3 - 60x^2y + 150xy^2$

$= (2x)^3 - (5y)^3 - 3(2x)(5y)(2x-5y)$

$= (2x-5y)^3$

Q 6 (i)

Use formula

$a^3+b^3 = (a+b)(a^2 - ab + b^2)$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

$27 + 8x^3$

$= (3)^3 + (2x)^3$

$= (3+2x)\{(3)^2 - (3)(2x) + (2x)^2\}$

$= (3+2x)(9 - 6x + 4x^2)$

ii) $125x^3 - 216y^3$

$= (5x)^3 - (6y)^3$

$= (5x-6y)[(5x)^2 + (5x)(6y) + (6y)^2]$

$= (5x-6y)(25x^2 + 30xy + 36y^2)$

Q 6 (iii)

$64x^3 + 27y^3$

$= (4x)^3 + (3y)^3$

$= (4x+3y)[(4x)^2 - (4x)(3y) + (3y)^2]$

$= (4x+3y)[16x^2 - 12xy + 9y^2]$

iv) $8x^3 + 125y^3$

$= (2x)^3 + (5y)^3$

$= (2x+5y)[(2x)^2 - (2x)(5y) + (5y)^2]$

$= (2x+5y)(4x^2 - 10xy + 25y^2)$

Exercise 5.3

1 (i) Use the remainder theorem to find the remainder

$3x^3 - 10x^2 + 13x - 6$ is divided by $(x-2)$

$P(x) = 3x^3 - 10x^2 + 13x - 6$

If we divide by $x-2$ then we will put $x = 2$

$P(2) = 3(2)^3 - 10(2)^2 + 13(2) - 6$

$= 3(8) - 10(4) + 26 - 6$

$= 24 - 40 + 26 - 6$

$P(2) = 4$

$\therefore$ Remainder is 4 when divided by $x-2$

Q1 ii) $4x^3 - 4x + 3$ divided by $(2x-1)$

Put $2x - 1 = 0$

$2x = 1$

$\boxed{x = \frac{1}{2}}$

$P(x) = 4x^3 - 4x + 3$

$P(\frac{1}{2}) = 4(\frac{1}{2})^3 - 4(\frac{1}{2}) + 3$

$= 4 \times \frac{1}{8_2} - 2 + 3$

$= \frac{1}{2} + 1$

$P(\frac{1}{2}) = 1\frac{1}{2} = \boxed{\frac{3}{2}}$

iii) $6x^4 + 2x^3 - x + 2$ divided by $(x+2)$

Put $x + 2 = 0$

$\boxed{x = -2}$

$P(x) = 6x^4 + 2x^3 - x + 2$

$= 6(-2)^4 + 2(-2)^3 - (-2) + 2$

$= 6(16) + (2)(-8) + 2 + 2$

$= 96 - 16 + 4$

$P(x) = 84$

iv) $(2x-1)^3 + 6(3+4x)^2 - 10$ divided by $2x + 1$

Put $2x + 1 = 0$

$2x = -1$

$\boxed{x = -\frac{1}{2}}$

$P(x) = (2x-1)^3 + 6(3+4x)^2 - 10$

$= \left[2(-\frac{1}{2}) - 1\right]^3 + 6\left[3 + 4(-\frac{1}{2})\right]^2 - 10$

$= [-1-1]^3 + 6[3-2]^2 - 10$

$= (-2)^3 + 6(1)^2 - 10$

$= -8 + 6(1) - 10$

$= -8 + 6 - 10$

$\boxed{P(x) = -12}$

(v) $x^3 - 3x^2 + 4x - 14$ is divided by $(x+2)$

Put $x + 2 = 0$

$\boxed{x = -2}$

$P(x) = x^3 - 3x^2 + 4x - 14$

$P(-2) = (-2)^3 - 3(-2)^2 + 4(-2) - 14$

$= -8 - 3(4) - 8 - 14$

$= -8 - 12 - 8 - 14$

$P(-2) = -42$

$\therefore$ Remainder is -42

Remainder theorem: If a Polynomial $P(x)$ is divided by a linear divisor $(x-a)$, then the remainder is $P(a)$

Q2 (i) If $(x+2)$ is a factor of $3x^2 - 4kx - 4k^2$ then find the values of (k)

Now put $x+2=0$

$\boxed{x=-2}$

$\therefore$

$P(x) = 3x^2 - 4kx - 4k^2$

$P(-2) = 3(-2)^2 - 4k(-2) - 4k^2$

$P(-2) = 3(4) + 8k - 4k^2$

$= 12 + 8k - 4k^2$

because $x+2$ is a factor

$\therefore$ $P(-2) = 0$

So $12 + 8k - 4k^2 = 0$

divide by 4

$\frac{\cancel{12}^{3}}{\cancel{4}} + \frac{\cancel{8}^{2}k}{\cancel{4}} - \frac{\cancel{4}k^2}{\cancel{4}} = \frac{0}{4}$

$3 + 2k - k^2 = 0$

Product $3 \times (-1) = -3$

diff $3 - 1 = 2$

$3 + 3k - k - k^2 = 0$

$3(1+k) - k(1+k) = 0$

$(3-k)(1+k) = 0$

$3-k=0$ | $1+k=0$

$\boxed{k=3}$ | $\boxed{k=-1}$

(ii) If $(x-1)$ is a factor of $x^3 - kx^2 + 11x - 6$ Then find the value of k

Put $x-1=0$

$x=1$

$P(x) = x^3 - kx^2 + 11x - 6$

$P(1) = (1)^3 - k(1)^2 + 11(1) - 6$

$P(1) = 1 - k + 11 - 6$

$P(1) = -k + 6$

$P(1) = 0$ because $(x-1)$ is a factor of $(P(x))$

$\therefore$ $-k + 6 = 0$

$\boxed{k=6}$

3/ i/ without actual long division determine whether $(x-2)$ and $(x-3)$ are factors of

$P(x) = x^3 - 12x^2 + 44x - 48$

First put $x=2$

$P(2) = (2)^3 - 12(2)^2 + 44(2) - 48$

$= 8 - 12(4) + 88 - 48$

$= 8 - 48 + 88 - 48$

$P(2) = 96 - 96 = 0$

because $P(2) = 0$

$\therefore$ $x-2$ is a factor of $P(x)$

Q3/ (i)

Now Put $x = 3$

$P(x) = x^3 - 12x^2 + 44x - 48$

$P(3) = (3)^3 - 12(3)^2 + 44(3) - 48$

$= 27 - 12(9) + 132 - 48$

$= 27 - 108 + 132 - 48$

$= 159 - 156$

$P(3) = 3 \neq 0$

$\therefore$ $(x-3)$ is not a factor of $P(x)$

Q3/ (ii) $(x-2)$; $(x+3)$ and $(x-4)$ are factors of $q(x)$

$q(x) = x^3 + 2x^2 - 5x - 6$

Put $\boxed{x = 2}$

$q(2) = (2)^3 + 2(2)^2 - 5(2) - 6$

$= 8 + 2(4) - 10 - 6$

$= 8 + 8 - 10 - 6$

$q(2) = \cancel{16} - \cancel{16} = 0$

- Yes $x-2$ is a factor of $q(x)$ because $q(2) = 0$

Now Put $\boxed{x = -3}$

$q(-3) = (-3)^3 + 2(-3)^2 - 5(-3) - 6$

$= -27 + 2(9) + 15 - 6$

$= -33 + 18 + 15$

$q(-3) = -\cancel{33} + \cancel{33} = 0$

Yes $x+3$ is a factor of $q(x)$

for $(x-4)$ is factor are not

Put $x = 4$

$q(4) = (4)^3 + 2(4)^2 - 5(4) - 6$

$= 64 + 2(16) - 20 - 6$

$= 64 + 32 - 26$

$= 96 - 26$

$q(4) = 70 \neq 0$

$\therefore$ $x-4$ is not a factor of $q(x)$

$\therefore$ $q(4) \neq 0$

4/ For what value of m is the Polynomial $P(x) = 4x^3 - 7x^2 + 6x - 3m$ exactly divisible by $x+2$?

Put $\boxed{x = -2}$

$P(x) = 4x^3 - 7x^2 + 6x - 3m$

$P(-2) = 4(-2)^3 - 7(-2)^2 + 6(-2) - 3m$

$= 4(-8) - 7(4) - 12 - 3m$

$= -32 - 28 - 12 - 3m$

$P(-2) = -72 - 3m$

because $x+2$ is a factor

$\therefore$ $P(-2) = 0$

So $-72 - 3m = 0$

$-72 = 3m$

$\frac{\cancel{-72}^{-24}}{\cancel{3}} = m$

$\boxed{m = -24}$

Q5/ Determine the value of K if $P(x) = Kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + K$ leaves the same remainder when divided by $(x-3)$.

Now Put $x = 3$

$$P(3) = K(3)^3 + 4(3)^2 + 3(3) - 4$$
$$= K(27) + 4(9) + 9 - 4$$
$$= 27K + 36 + 9 - 4$$
$$P(3) = 27K + 41 \quad \text{--- (1)}$$

$$q(3) = (3)^3 - 4(3) + K$$
$$q(3) = 27 - 12 + K$$
$$q(3) = 15 + K \quad \text{--- (2)}$$

because given that

$q(3) = P(3)$ {(Same remainder) from (1) and (2)

$$27K + 41 = 15 + K$$
$$27K - K = 15 - 41$$
$$26K = -26$$
$$K = \frac{-\cancel{26}}{\cancel{26}} = -1$$

$$\boxed{K = -1}$$

Q6/ The remainder after dividing the Polynomial $P(x) = x^3 + ax^2 + 7$ by $(x+1)$ is $2b$. Calculate the value of a and b if this expression leaves a remainder of $b+5$ on being divided by $(x-2)$

Sol.

$$P(x) = x^3 + ax^2 + 7$$

First Put $x = -1$

$$P(-1) = (-1)^3 + a(-1)^2 + 7$$
$$= -1 + a(1) + 7$$
$$P(-1) = a + 6$$

given that $P(-1) = 2b$.

So $a + 6 = 2b$ --- (1)

$$\boxed{a = 2b - 6}$$

Now Put $x = 2$

$$P(2) = (2)^3 + a(2)^2 + 7$$
$$P(2) = 8 + 4a + 7$$
$$P(2) = 4a + 15$$

given that $P(2) = b + 5$

$$b + 5 = 4a + 15 \quad \text{--- (2)}$$

Now Put value of "a" in Eq (2) from (1) eq

$$b + 5 = 4(2b - 6) + 15$$
$$b + 5 = 8b - 24 + 15$$
$$24 + 5 - 15 = 8b - b$$
$$14 = 7b \Rightarrow \boxed{b = 2}$$

Q6/ Remaining Part

Put value of $b = 2$ in Eq (1)

$$a = 2(2) - 6$$
$$= 4 - 6 = -2$$

Q7/ The polynomial $x^3+lx^2+mx+24$ has a factor $(x+4)$ and it leaves a remainder of 36 when divided by $(x-2)$. Find the values of "l" and "m".

Sol. Put $\boxed{x=-4}$

$P(x) = x^3+lx^2+mx+24$

$P(-4) = (-4)^3 + l(-4)^2 + m(-4) + 24$

$P(-4) = -64 + 16l - 4m + 24$

$P(-4) = 16l - 4m - 40$

because $(x+4)$ is a factor of $P(x)$ so $P(-4)=0$

$\therefore \quad 16l - 4m - 40 = 0$ —— (1)

Now Put $\boxed{x=2}$

$P(x) = x^3 + lx^2 + mx + 24$

$P(2) = (2)^3 + l(2)^2 + m(2) + 24$

$P(2) = 8 + 4l + 2m + 24$

$P(2) = 4l + 2m + 32$

$\because$ because remainder is 36

So $P(2) = 36$

$\therefore \quad 4l + 2m + 32 = 36$

$4l + 2m + 32 - 36 = 0$

$4l + 2m - 4 = 0$ —— (2)

Now multiply Eq (2) by (2) and add them

$2(4l + 2m - 4) = 0$

$8l + 4m - 8 = 0$

$$\begin{array}{r} 16l - \cancel{4m} - 40 = 0 \\ 8l + \cancel{4m} - 8 = 0 \\ \hline 24l \quad\quad - 48 = 0 \end{array}$$

$24l = 48$

$l = 48/24 = 2$

$\Rightarrow \boxed{l=2}$

Now put this value in Eq 2

$4(2) + 2m - 4 = 0$

$8 + 2m - 4 = 0$

$2m + 4 = 0$

$2m = -4$

$m = -\cancel{4}^{-2}/\cancel{2}$

$\boxed{m=-2}$

8/ The expression lx^3+mx^2-4 leaves remainder -3 and 12 when divided by $(x-1)$ and $(x+2)$ respectively. Calculate the values of l and m.

Put $x=1$

$P(x) = lx^3 + mx^2 - 4$

$P(1) = l(1)^3 + m(1)^2 - 4$

$P(1) = l + m - 4$

given that $P(1) = -3$

$\therefore \quad l + m - 4 = -3$

$l + m = -3 + 4$

$\boxed{l + m = 1}$ —— (1)

8/ Remaining Part

$P(x) = l x^3 + mx^2 - 4$

Put $\boxed{x = -2}$

$P(-2) = l(-2)^3 + m(-2)^2 - 4$

$= -8l + m(4) - 4$

$P(-2) = -8l + 4m - 4$

given that

$P(-2) = 12$

$\therefore$ $-8l + 4m - 4 = 12$

$-8l + 4m = 12 + 4$

$-8l + 4m = 16$ —— (2)

multiply Eq (1) by 8 and add in Eq (2) we have

$-\cancel{8l} + 4m = 16$

$+\cancel{8l} + 8m = 8$

$12m = 24$

$m = \cancel{24}^{2}/\cancel{12}$

$\boxed{m = 2}$

Now for "l" Put $m = 2$ in Eq (1) we have

$l + m = 1$

$l + 2 = 1$

$l = 1 - 2 = -1$

$\boxed{l = -1}$

9/ The expression $ax^3 - 9x^2 + bx + 3a$ is exactly divisible by $x^2 - 5x + 6$. Find values of "a" and "b".

Sol/ First factorize $(x^2 - 5x + 6$

$x^2 - 3x - 2x + 6$

$= x(x-3) - 2(x-3)$

$= (x-2)(x-3)$

Product
$1 \times 6 = 6$
$3 \times 2 = 6$
Sum
$-3 - 2 = -5$

Now for factor $x-2$

Put $\boxed{x = 2}$

$P(x) = ax^3 - 9x^2 + bx + 3a$

$P(2) = a(2)^3 - 9(2)^2 + b(2) + 3a$

$P(2) = 8a - 36 + 2b + 3a$

$P(2) = 11a + 2b - 36$

$P(2) = 0$ because $(x-2)$ is a factor

$11a + 2b - 36 = 0$ —— (1)

Now Put $\boxed{x = 3}$

$P(3) = a(3)^3 - 9(3)^2 + b(3) + 3a$

$= 27a - 9(9) + 3b + 3a$

$P(3) = 30a + 3b - 81$

$P(3) = 0$ because $(x-3)$ is a factor of $P(x)$

$30a + 3b - 81 = 0$ —— (2)

multiply Eq (2) by 2 and Eq (1) by 3 and then subtract Eq (1) from (2)

$60a + \cancel{6b} - 162 = 0$

$-33a + \cancel{6b} - 108 = 0$

$-\quad\quad\quad -\quad\quad +$

$27a \quad - 54 = 0$

$27a = 54$

$\boxed{a = 2}$

9/ remaining part.

Now Put $a=2$ in Eq(1)

we have

$11a+2b-36=0$

$11(2)+2b-36=0$

$2b-36+22=0$

$2b-14=0$

$2b=14$

$\boxed{b=7}$

Exercise 5-4

1/ We use the hit and trial method to find zeros of $P(x)$. Let us try $x=1$ in $P(x)=x^3-2x^2-x+2$

$P(1)=(1)^3-2(1)^2-1+2$

$=\cancel{1}-\cancel{2}-\cancel{1}+\cancel{2}$

$P(1)=0$

$\therefore$ $x=1$ is a root

and $x-1$ is a factor of $P(x)$

Now use Synthetic division Method to find other factors

1	1	-2	-1	2
		1	-1	-2
	1	-1	-2	0

$\therefore$ x^2-x-2 is a Quotient

$\therefore$

$x^3-2x^2-x+2=(x-1)(x^2-x-2)$

$P(x)=(x-1)\left[x^2-2x+x-2\right]$

$P(x)=(x-1)\left[x(x-2)+1(x-2)\right]$

$P(x)=(x-1)\left[(x-2)(x+1)\right]$

$P(x)=(x-1)(x-2)(x+1)$

2/ $P(x)=x^3-x^2-22x+40$

Try $x=1$

$P(1)=(1)^3-(1)^2-22(1)+40$

$=\cancel{1}-\cancel{1}-22+40$

$P(1)=18\neq 0$

$\therefore$ $x=1$ is not zero of $P(x)$

$\therefore$ we will Now Try $x=2$

$P(2)=(2)^3-(2)^2-22(2)+40$

$=8-4-44+40$

$P(2)=48-48=0$

$\therefore$ $x=2$ is a zero of $P(x)$

so $x-2$ is factor of $P(x)$

2	1	-1	-22	40
		2	2	-40
	1	1	-20	0

$\therefore$ x^2+x-20 is a Quotient

Exercise 5.4

$P(x) = (x-2)(x^2+x-20)$

$= (x-2)\left[x^2+5x-4x-20\right]$

$= (x-2)\left[x(x+5)-4(x+5)\right]$

$= (x-2)\left[(x+5)(x-4)\right]$

$P(x) = (x-2)(x+5)(x-4)$

3/ $P(x) = x^3-6x^2+3x+10$

Try $x=2$

$P(2) = (2)^3-6(2)^2+3(2)+10$

$= 8-6(4)+6+10$

$P(2) = \cancel{24}-\cancel{24} = 0$

So $x=2$ is a root of $P(x)$

$$\begin{array}{c|ccc|c} 2 & 1 & -6 & 3 & 10 \\ & & 2 & -8 & -10 \\ \hline & 1 & -4 & -5 & 0 \end{array}$$

$\therefore$ x^2-4x-5 is a Quotient

$P(x) = (x-2)(x^2-4x-5)$

$= [x-2]\left[x^2-5x+x-5\right]$

$= (x-2)\left[x(x-5)+1(x-5)\right]$

$P(x) = (x-2)(x+1)(x-5)$

are factors of $P(x)$

4/ $P(x) = x^3+x^2-10x+8$

Try $x=1$

$P(1) = (1)^3+(1)^2-10(1)+8$

$= 1+1-10+8$

$P(1) = \cancel{10}-\cancel{10} = 0$

$\therefore$ $x=1$ is a root of $P(x)$ and $x-1$ is a factor of $P(x)$

$$\begin{array}{c|ccc|c} 1 & 1 & 1 & -10 & 8 \\ & & 1 & 2 & -8 \\ \hline & 1 & 2 & -8 & 0 \end{array}$$

$\therefore$ x^2+2x-8 is a Quotient

So $P(x) = (x-1)(x^2+2x-8)$

$= (x-1)\left[x^2+4x-2x-8\right]$

$= (x-1)\left[x(x+4)-2(x+4)\right]$

$P(x) = (x-1)(x+4)(x-2)$

Zero of a Polynomial:

If a specific number $x=a$ is substituted for the variable "x" in a polynomial $P(x)$. So that the value $P(a)$ is zero, then $x=a$ is called zero of $P(x)$.

5/ $P(x) = x^3 - 2x^2 - 5x + 6$

Try $x = 1$

$P(1) = (1)^3 - 2(1)^2 - 5(1) + 6$

$= 1 - 2 - 5 + 6$

$P(1) = \cancel{7} - \cancel{7} = 0$

$\therefore$ $x = 1$ is a zero of $P(x)$ and $(x-1)$ is a factor of $P(x)$

1	1	-2	-5	6
		1	-1	-6
	1	-1	-6	0

$\therefore$ $x^2 - x - 6$ is a Quotient

So

$P(x) = (x-1)(x^2 - x - 6)$

$= (x-1)\left[x^2 - 3x + 2x - 6\right]$

$= (x-1)\left[x(x-3) + 2(x-3)\right]$

$= (x-1)(x-3)(x+2)$

6/ $P(x) = x^3 + 5x^2 - 2x - 24$

Try $x = 2$

$P(2) = (2)^3 + 5(2)^2 - 2(2) - 24$

$= 8 + 5(4) - 4 - 24$

$= 8 + 20 - 28$

$P(2) = \cancel{28} - \cancel{28} = 0$

$\therefore$ $x = 2$ is zero of $P(x)$

and $x - 2$ is a factor of $P(x)$

2	1	5	-2	-24
		2	14	24
	1	7	12	0

$\therefore$ $x^2 + 7x + 12$ is a Quotient or the factor of $P(x)$

So

$P(x) = (x-2)(x^2 + 7x + 12)$

$= (x-2)\left[x^2 + 4x + 3x + 12\right]$

$= (x-2)\left[x(x+4) + 3(x+4)\right]$

$= (x-2)(x+4)(x+3)$

7/ $P(x) = 3x^3 - x^2 - 12x + 4$

Try $x = 2$

$P(2) = 3(2)^3 - (2)^2 - 12(2) + 4$

$= 3(8) - 4 - 24 + 4$

$= 24 - 28 + 4$

$P(2) = \cancel{28} - \cancel{28} = 0$

$P(2) = 0$

$\therefore$ $x = 2$ is a zero of $P(x)$

2	3	-1	-12	4
		6	10	-4
	3	5	-2	0

$\therefore$ $3x^2 + 5x - 2$ is a Quotient or factor of $P(x)$

7/ remaining Part

$P(x) = (x-2)(3x^2+5x-2)$

$= (x-2)[3x^2+6x-x-2]$

$= (x-2)[3x(x+2)-1(x+2)]$

$= (x-2)(3x-1)(x+2)$

8/

$P(x) = 2x^3+x^2-2x-1$

Try $x=1$

$p(1) = 2(1)^3+(1)^2-2(1)-1$

$= 2+1-2-1$

$P(1) = \cancel{3}-\cancel{3}$

$= 0$

$\therefore$ $x=1$ is a zero of $P(x)$ and $(x-1)$ is a factor of $P(x)$

$$\begin{array}{c|cccc}
1 & 2 & 1 & -2 & \cancel{-1} \\
 & & 2 & 3 & \cancel{1} \\
\hline
 & 2 & 3 & 1 & 0
\end{array}$$

$\therefore$ $2x^2+3x+1$ is a Quotient or factor of $P(x)$

$\therefore$ $P(x) = (x-1)(2x^2+3x+1)$

$= (x-1)[2x^2+2x+x+1]$

$= (x-1)[2x(x+1)+1(x+1)]$

$p(x) = (x-1)(2x+1)(x+1)$

Review Exercise

3/ (i) factorize the following

$x^2+8x+16-4x^2$

$= (x)^2+2(x)(4)+(4)^2-4x^2$

$= (x+4)^2-4x^2$

$= (x+4)^2-(2x)^2$

$= (x+4+2x)(x+4-2x)$

ii) $4x^2-16y^2$

$= (2x)^2-(4y)^2$

$= (2x+4y)(2x-4y)$

iii)

$9x^2+27x+8$

$9x^2+24x+3x+8$

$3x(3x+8)+1(3x+8)$

$(3x+1)(3x+8)$

Product
$9 \times 8 = 72$
$24 \times 3 = 72$
Sum
$24+3 = 27$

(iv)

$1-64z^3$

$(1)^3-(4z)^3$

$= (1-4z)[(1)^2+(1)(4z)+(4z)^2]$

$= (1-4z)(1+4z+16z^2)$

formula is
$a^3-b^3 = (a-b)(a^2+ab+b^2)$

Review Exercise 5. P#112

3/ (v) $8x^3 - \frac{1}{27y^3}$

$8x^3 - \frac{1}{27y^3} = (2x)^3 - \frac{1}{(3y)^3}$

$= \left(2x - \frac{1}{3y}\right)\left[(2x)^2 + (2x)\left(\frac{1}{3y}\right) + \left(\frac{1}{3y}\right)^2\right]$

$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$

(vi)

$2y^2 + 5y - 3$

$= 2y^2 + 6y - y - 3$

$= 2y(y+3) - 1(y+3)$

$= (2y-1)(y+3)$

Product
$2 \times (-3) = -6$
$6 \times 1 = 6$
diff
$6 - 1 = 5$

vii) $x^3 + x^2 - 4x - 4$

$= x^2(x+1) - 4(x+1)$

$= (x^2 - 4)(x+1)$

$= [(x)^2 - (2)^2](x+1)$

$= (x+2)(x-2)(x+1)$

viii) $25m^2n^2 + 10mn + 1$

$= (5mn)^2 + 2(5mn)(1) + (1)^2$

$= (5mn + 1)^2$

IX) $1 - 12pq + 36p^2q^2$

$= (1)^2 - 2(1)(6pq) + (6pq)^2$

$= (1 - 6pq)^2$

good luck